Optimal Cable Positioning

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July 23, 2017

Abstract

Looking at the suggested designs given by Technip, we can see that there seems to be no limit to the number of positions that the cables can occupy within the umbilical. Clearly as we can add any number of fillers of any size, there is not a finite number of cable configurations. So it is not possible to just check all viable cable configurations one by one. Instead, we need to allow, mathematically, these cables to be placed any where as long as they do not overlap or break any other given restriction. Naturally this leads us to formulate the problem as an optimisation problem with continuous variables.

Keywords: optimised design, umbilical cables, optimisation.

1 Describing the problem mathematically

Here we translate from English to mathematics the several desirable properties of an optimised umbilical design.

We begin by specifying an euclidean (x, y) coordinate system, where $(0, 0)$ is the centre of the umbilical. We will refer to all the possible conduits, cables and tubes as just cables. For simplicity, say we want the umbilical to contain two steel cables and two quad cables with radius R^S and R^Q, respectively. We describe the position of the centre of the steel cables by $\boldsymbol{X}_1^S = (x_1^S, y_1^S)$ and $\boldsymbol{X}_2^S = (x_2^S, y_2^S)$, and the centre of quad cables by $\boldsymbol{X}_1^Q = (x_1^Q, y_1^Q)$ and $\boldsymbol{X}_2^Q = (x_2^Q, y_2^Q)$. See Figure 1 for an illustration.

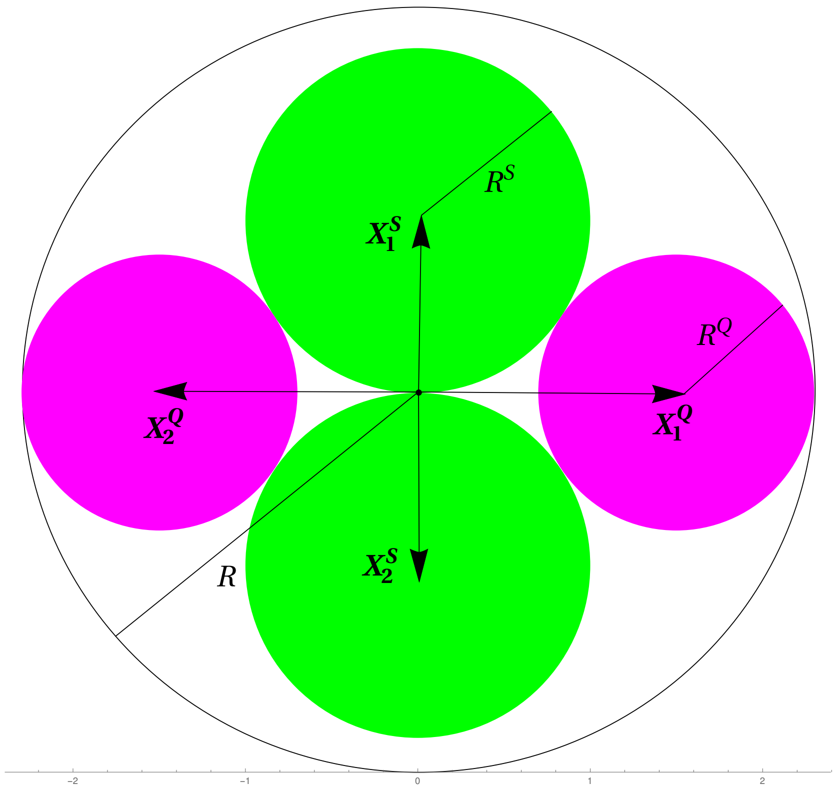
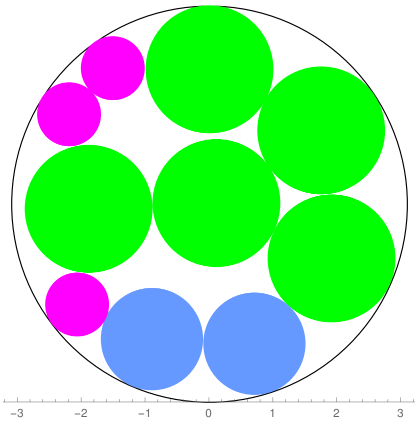
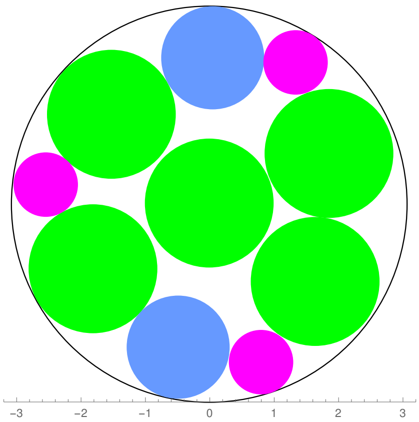


Figure 1: the position of a cable is described in terms of the vector to its centre and its radius.

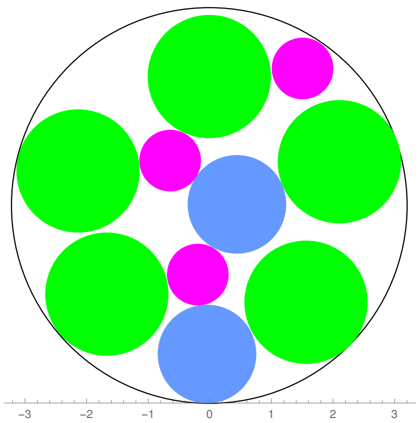
We can now translate the desirable features of an umbilical to simple mathematical expressions. Figure 2 shows how the different costs described below influence the optimised design.



(**a**) Smallest area



(**b**) Balance stiffness



(**c**) Avoid steel touching

Figure 2: Shows how the results of an optimisation method change when adding extra cost functions. **a**) only consider the cost of the area ($C_{area} = 1$ and $C_{steel} = C_{stiff} = 0$); **b**) also includes the cost of balancing the stiffness ($C_{area} = C_{stiff} = 1$ and $C_{steel} = 0$); **c**) further includes the cost of steel cables touching ($C_{area} = C_{stiff} = C_{steel} = 1$). Note that due to their lack of radial symmetry *these results can not be manufactured*, something which is corrected later and the improved results presented in section 3.

1.1 Estimating cost

We begin by assuming the cost of fabrication is proportional to the area of the umbilical[1], so that there is an added cost of

$$C_{area}R^2, \tag{1}$$

[1] The cost of having a two assembly design is not considered, but could have easily been included.

where R is the radius of the umbilical and C_{area} is the cost per unit area.

In certain situations, it is undesirable for the steel cables to touch. We can assume that when the steel cables do touch, they wear away more quickly and this incurs an added long term cost C_{steel}. A simple, smooth function that only adds cost when the steel cables touch is

$$C_{steel} f \left(\left(2R^S\right)^2 - \|\boldsymbol{X}_1^S - \boldsymbol{X}_2^S\|^2 \right), \tag{2}$$

where $\|(x, y) - (w, z)\| := \sqrt{(x-w)^2 + (y-z)^2}$, f is a sigmoid function such as $f(t) = (1 + \mathrm{e}^{-20t})$, and we used a power of two so that this cost is a smooth function of its variables (an important feature in gradient based nonlinear optimisation).

We can also include a design rule of thumb: stiffer cables (i.e. steel cables) should be well distributed throughout the umbilical. We can again assume that a long term cost C_{stiff} is incurred if the stiffer cables are badly distributed. Say that steel and quad cable have a stiffness of S_{steel} and S_{quad}. We can use principals of physics such as centre of mass or centre of inertia (when applying torque to the boundary of the umbilical), but with the stiffness's S_{steel} and S_{quad} instead of the masses. The simplest of these is the centre of stiffness:

$$C_{stiff} \|S_{steel} \boldsymbol{X}_1^S + S_{steel} \boldsymbol{X}_2^S + S_{quad} \boldsymbol{X}_1^Q + S_{quad} \boldsymbol{X}_2^Q\|^2. \tag{3}$$

The above formula gives zero when the centre of stiffness is at the centre of the umbilical, otherwise a cost is incurred.

1.2 Constraints

The cables can neither overlap, nor can they be outside the umbilical casing. For the cables to not overlap translates to

$$\begin{aligned}
&\|\boldsymbol{X}_1^S - \boldsymbol{X}_2^S\|^2 - \left(2R^S\right)^2 \geq 0, &&\|\boldsymbol{X}_1^S - \boldsymbol{X}_1^Q\|^2 - \left(R^S + R^Q\right)^2 \geq 0,\\
&\|\boldsymbol{X}_1^S - \boldsymbol{X}_2^Q\|^2 - \left(R^S + R^Q\right)^2 \geq 0, &&\|\boldsymbol{X}_2^S - \boldsymbol{X}_1^Q\|^2 - \left(R^S + R^Q\right)^2 \geq 0,\\
&\|\boldsymbol{X}_2^S - \boldsymbol{X}_2^Q\|^2 - \left(R^S + R^Q\right)^2 \geq 0, &&\|\boldsymbol{X}_1^Q - \boldsymbol{X}_2^Q\|^2 - \left(2R^Q\right)^2 \geq 0. \qquad (4)
\end{aligned}$$

A trick often used in optimisation is to turn a constraint into a cost penalisation. For this penalisation to be a smooth function of its variables, we again used the power of two in the constraint.

Forcing the cables to be within the umbilical casing leads to the constraint

$$\begin{aligned}
&\left(R - R^S\right)^2 - \|\boldsymbol{X}_1^S\|^2 \geq 0, &&\left(R - R^S\right)^2 - \|\boldsymbol{X}_2^S\|^2 \geq 0,\\
&\left(R - R^Q\right)^2 - \|\boldsymbol{X}_1^Q\|^2 \geq 0, &&\left(R - R^Q\right)^2 - \|\boldsymbol{X}_2^Q\|^2 \geq 0. \qquad (5)
\end{aligned}$$

for a two assembly design we need to add another constraint for the components added in the second assembly. Let R^0 be the radius of the first assembly, and assume we want to place both our steel and quad cables in the second assembly process, then

$$\begin{aligned}
&R - R^0 - 2R^S \geq 0, &&R - R^0 - 2R^Q \geq 0, \qquad (6)\\
&(R^0 + R^S)^2 - \|\boldsymbol{X}_1^S\|^2 \leq 0, &&(R^0 + R^Q)^2 - \|\boldsymbol{X}_1^Q\|^2 \leq 0,\\
&(R^0 + R^S)^2 - \|\boldsymbol{X}_2^S\|^2 \leq 0, &&(R^0 + R^Q)^2 - \|\boldsymbol{X}_2^Q\|^2 \leq 0. \qquad (7)
\end{aligned}$$

The first inequality (6) states that each of the remaining cables fit between the inner and outer layer. The following inequalities (7) stop any of the remaining cables from overlaping with the inner radius.

2 General considerations

There are many optimisation methods, packages and programmes available. Instead of describing one particular approach, we will outline some general considerations for all gradient based methods. Later in Section 3 we show some results.

2.1 Two assembly process and radial symmetry

To actually build any design it needs to poses enough radial symmetry near the centre, see Figure 2 for examples of designs which can not be built. We can remedy this by spilting the method into two steps: first choose a small set of cables (with at least one cable) to go in the centre of the umbilical, then run the optimisation method just for these cables. For the next step let R^0 be the radius of the smallest circle that contains the cables so far. With this first set of cables fixed, now run the optimisation algorithm for the remain cables with the added constraints (6) and (7).

Splitting the method in two steps guarantees radial symmetry in the centre of the umbilical. If only one cable is placed in the centre then we can interpret this as a one assembly process, whereas if two or more cables are placed in the centre then the design is a two assembly process.

2.2 Choosing an initial cable configuration

Putting all the constrains together can leave little room for the cables to move and rearrange. It easy for an optimisation method to get stuck in one particular cable configuration (a local minimum). This is why it is important to have many different initial values for the method, that should ideally be well distributed. Then, the method can be run separately for each of these initial values and compare the results. We achieve this by generating a large set of random initial cable configurations, and then remove the cable configuration which are similar.

In the next section we show in more detail how to measure if two cable configurations are similar, as this technique is also useful when choosing between optimised designs.

2.3 Measuring similar cable configurations

How do we measure if two cable configurations are alike, in the sense that they would give umbilicals with the same mechanical properties?

Let all the positions of the centre of the cables be denoted by $\boldsymbol{X}_1, \boldsymbol{X}_2 \ldots \boldsymbol{X}_N$. Create a matrix $\boldsymbol{M}$, where $M_{ij} = \boldsymbol{X}_i \cdot \boldsymbol{X}_j$ and $(x, y) \cdot (z, w) := xz + yw$. This way a rotation of all the $\boldsymbol{X}_i$'s will not affect $\boldsymbol{M}$.

Now the order which we list the cables $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots$ should also not influence our measure. Note that swapping the order of two cables, for example swapping $\boldsymbol{X}_1$ with $\boldsymbol{X}_2$, is equivalent to swapping $\boldsymbol{M}$ for $\boldsymbol{PMP}$ where $\boldsymbol{P}$ is a row-switching matrix with the properties $\boldsymbol{PP} = \boldsymbol{I}$ and $\det \boldsymbol{P} = -1$. One way to eliminate the influence of the order is to apply sevarel row-switching matrices until $(M_{11}, M_{22}, \ldots, M_{NN})$, the diagonal of $\boldsymbol{M}$, is in decreasing order. Yet another way is to combine the components of $\boldsymbol{M}$ to form quantities that are invariant to row-switching operations, such as the isotropic invariants

$$\boldsymbol{S}(\boldsymbol{M}) = \left(\operatorname{tr}(\boldsymbol{M}), \operatorname{tr}(\boldsymbol{M} \cdot \boldsymbol{M}), \ldots, \operatorname{tr}(\boldsymbol{M}^N)\right). \tag{8}$$

With the above, the matrices $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$ of two cable configurations can be compared by computing the difference $\|\boldsymbol{\mathcal{S}}(\boldsymbol{M}_1) - \boldsymbol{\mathcal{S}}(\boldsymbol{M}_2)\|$.

3 Results

Here we present the results of a gradient based method implemented in Mathematica 10, which is run with a large set of random initial cable configurations.

To illustrate, we choose the cables

- 5 steel cables with radius $R^S = 10\text{cm}$ and stiffness $S_{steel} = 10\text{cm}^{-2}$
- 2 quad cables with radius $R^Q = 9\text{cm}$ and stiffness $S_{quad} = 2\text{cm}^{-2}$
- 3 optical cables with radius $R^O = 5\text{cm}$ and stiffness $S_{optical} = 1\text{cm}^{-2}$

In section 1.1 we see that we have to choose three cost parameters C_{area}, C_{steel} and C_{stiff}. We pick two examples

$$\text{example 1:} \;\; C_{area} = 2C_{stiff} = \frac{1}{2}C_{steel}, \tag{9}$$

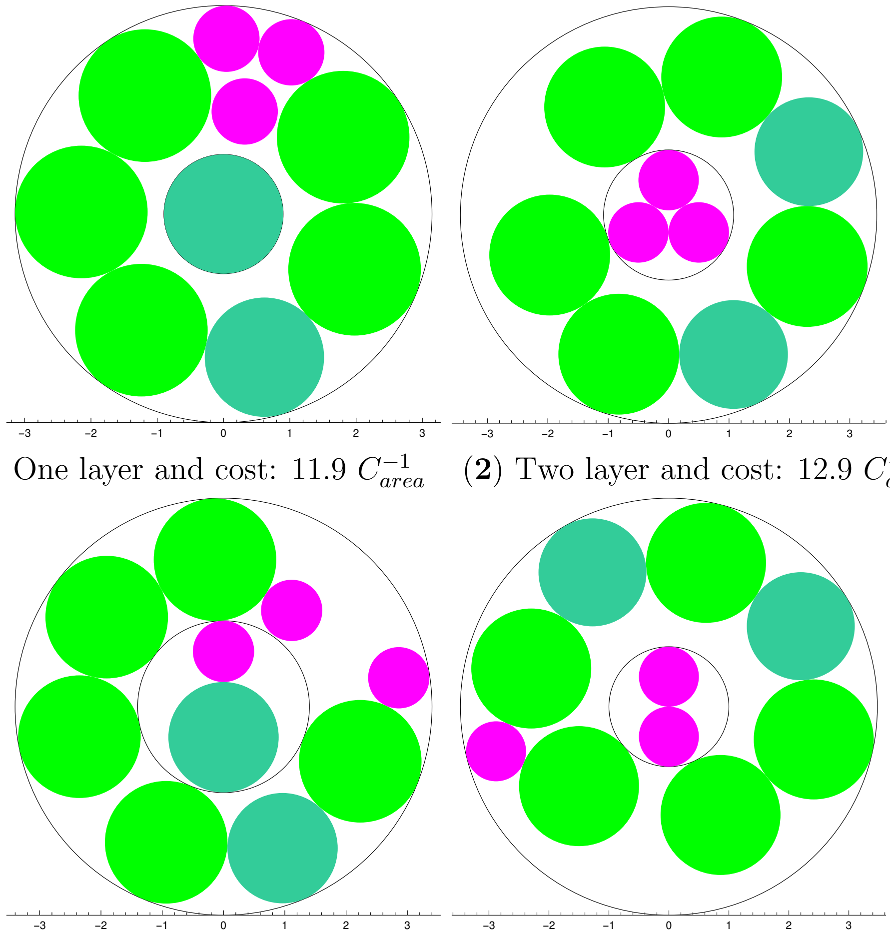
$$\text{example 2:} \;\; C_{area} = C_{stiff} \quad \text{and} \quad C_{steel} = 0, \tag{10}$$

where we do not give a value for C_{area} as only the ratio between the costs will effect the resulting optimal designs.

Example 1 means that it is more important that steel cables do not touch and least important is to balance the stiffness, see Figure 3 for the results. Example 2 gives equal importance to balancing the stiffness and reducing the area of the umbilical, while not considering if the steel cables touch. See Figure 4 for the results.

Remember we have not included the cost of having a two assembly process (two layers), nor do we consider any complications arising from adding fillers.

The method presented is simply a proof of concept, but could be extended to include all these considerations that have left out and more.



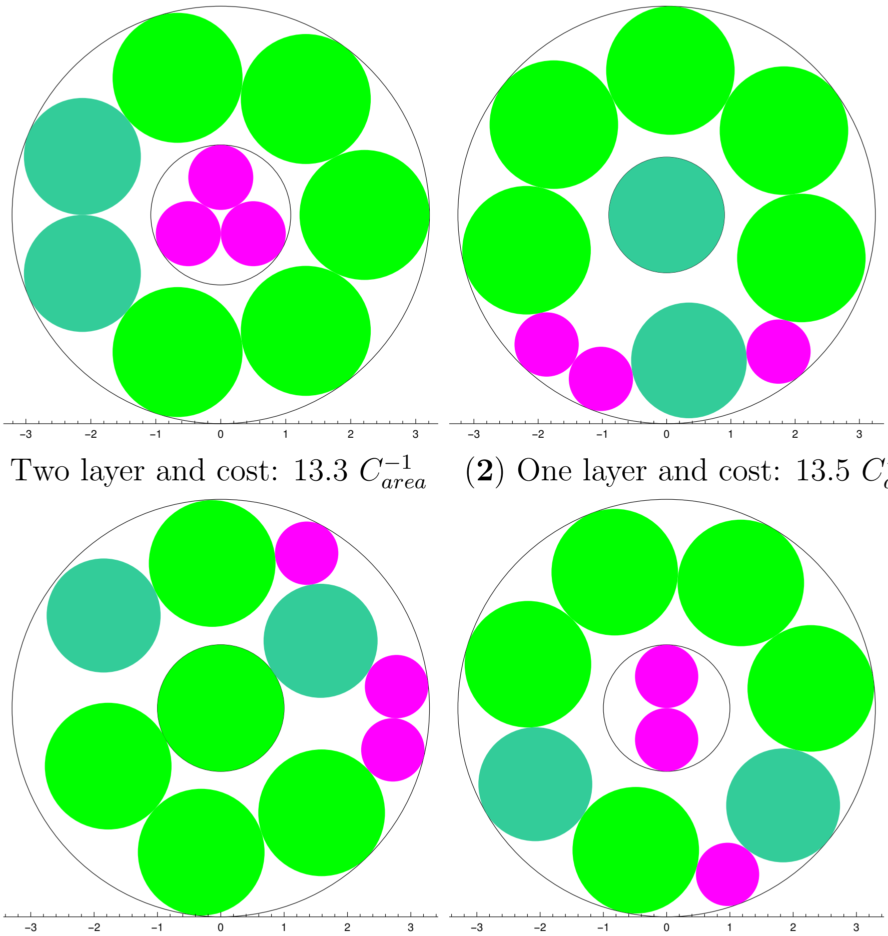
(**1**) One layer and cost: 11.9 C_{area}^{-1} (**2**) Two layer and cost: 12.9 C_{area}^{-1}

(**3**) Two layer and cost: 13.1 C_{area}^{-1} (**4**) Two layer and cost: 13.1 C_{area}^{-1}

Figure 3: Shows the optimal designs for the cables specified just above equation (9), which gives the cost parameters. The large light green circles are the steel cables, the small purple circles are the optical cables and the mid-sized jade circles are the quad cables. Note how in general the steel cables avoid touching and are well distributed in the umbilical (due to the balance of stiffness).

4 Conclusion

To summarise, this report showed how to translate several features of an optimised umbilcal into mathematics. We also proposed that we can calculate



(**1**) Two layer and cost: 13.3 C_{area}^{-1} (**2**) One layer and cost: 13.5 C_{area}^{-1}

(**3**) One layer and cost: 13.9 C_{area}^{-1} (**4**) Two layer and cost: 13.9 C_{area}^{-1}

Figure 4: Shows the optimal designs for the cables specified just above equation (10), which gives the cost parameters. The large light green circles are the steel cables, the small purple circles are the optical cables and the mid-sized jade circles are the quad cables. Note how in general these umbilicals are more compact than those given in Figure 3 because steel cables are allowed to touch.

the optimised designs by using nonlinear optimisation methods. As a proof of concept, we used Mathmatica's optimisation tools to generate the optimal designs shown in Figures 3 and 3.

Both the cost and stiffness parameters used in the method were not known, such as C_{Area} and S_{steel}. To improve the method, these parameters can be roughly estimated and then the method can search within these estimates to produce optimised designs.

The results shown here were achieved in only a few days of work, so we believe that we have clearly shown that it is possible to automatically produce optimal umbilical designs. Possible next steps to take this idea forward would be to hire the sevices of a mathematical consultancy, or enter a partnership with a academics from a computer science or applied mathematics departement. With a university, there is the option of funding a PhD student or post doctural researcher to work in collaboration with Technip.